

BEVEL GEAR AND WORM GEAR.

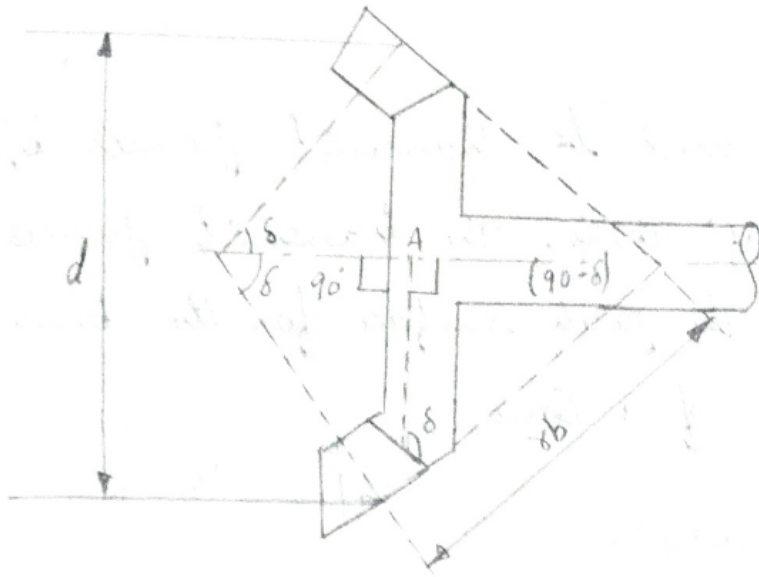
Bevel gear :- These are used to transmit power b/w two intersecting shaft axis, The transmit power @ constant velocity. The pitch surface for the bevel gear is frustum of a cone.

Classification of Bevel gears:-

The Bevel gear may be classified into following type depending upon the angle between the shaft and pitch surface.

1. miter gear :- when equal bevel gears (having equal teeth and equal pitch angle) connect two shafts whose axis intersect @ right angle then they are known as miter gear.
2. angular Bevel gear :- when the Bevel gear connects to shafts whose axis intersect @ an angle other than right angle they are known as angular Bevel gear.
3. Crown bevel gear :- when the bevel gear connects to shafts whose axis intersect @ an angle greater than right angle and one of the bevel gear has pitch angle of 90° . then it is known as Crown bevel gear.

Formation of equivalent number of teeth (virtual no of teeth)



r_b = back cone radius

δ = pitch cone angle

d = pitch circle dia.

From $\Delta^u ABC$

$$\sin(90 - \delta) = \frac{AB}{BC}$$

$$\sin(90 - \delta) = \frac{d/2}{r_b}$$

$$r_b = \frac{d/2}{\sin(90 - \delta)}$$

$$r_b = \frac{d}{2 \cos \delta}$$

Formative number of teeth are the teeth on equivalent spur gear having pitch cone diameter = Back cone radius.

Pitch Circle Equivalent diameter

$$d_e = 2 \times r_b$$

$$d_e = \cancel{2} \times \left(\frac{d}{\cancel{2} \cos \delta} \right)$$

$$d_e = \frac{d}{\cos \delta}$$

WKT. virtual number of teeth.

$$Z_e = \frac{d_e}{m} \Rightarrow Z_e = \frac{d}{m \cos \delta} \longrightarrow \textcircled{a}$$

In bevel gear, WKT.

$$Z = \frac{d}{m} \Rightarrow d = m \times Z \longrightarrow \textcircled{b}$$

substitute eqⁿ (b) in eqⁿ (a)

$$Z_e = \frac{mZ}{m \cos \delta} \Rightarrow Z_e = \frac{Z}{\cos \delta}$$

The tangential tooth load.

$$F_t = Vd \times C_v \times b \times Y \times m \left[\frac{L-b}{L} \right] \longrightarrow (\text{from eq. 12.37 pg 218})$$

Dynamic load equation.

$$F_d = F_t + \frac{K_3 V (C_b + F_t)}{K_3 V + \sqrt{C_b + F_t}} \longrightarrow (\text{from eq 12.12 pg 207})$$

wear load equation,

$$F_w = \frac{d_1 b d_c K}{\cos \delta_1}$$

where, $d_c = \frac{2 Z_e}{(Z_{e2} + Z_{e1})}$

Problems:-

1. Two shafts inclined @ 60° are connected by a pair of bevel gear to transmit 9kW @ 900rpm. of 24 teeth. CS pinion. The gear is made up of high grade C1. and is runs @ 300rpm. The teeth are 14.5° Involute from design the gear completely.

Given data:-

$$\theta = 60^\circ \text{ (acute angle).}$$

$$P = 9 \text{ kW}$$

$$n_1 = 900 \text{ rpm}$$

$$n_2 = 300 \text{ rpm}$$

$$Z_1 = 24 \text{ teeth}$$

$$\phi = 14.5^\circ$$

$$K = ?$$

$$\text{BHN} = ?$$

Solution:-

$$i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1} \Rightarrow \frac{900}{300} = \frac{Z_2}{24} \Rightarrow \boxed{Z_2 = 72 \text{ teeth}}$$

Pitch Cone angle for acute angle (δ):-

$$\tan \delta_1 = \frac{\sin \theta}{\frac{Z_2}{Z_1} + \cos \theta} \rightarrow (\text{eq 12.29 Pg 215})$$

$$\tan \delta_1 = \frac{\sin (60)}{\frac{72}{24} + \cos (60)}$$

$$\boxed{\delta_1 = 13.89^\circ}$$

$$\tan \delta_2 = \frac{\sin \theta}{\frac{Z_1}{Z_2} + \cos \theta} \rightarrow (\text{eq 12.29 (b) Pg 215})$$

$$\tan \delta_2 = \frac{\sin 60}{\frac{24}{72} + \cos (60)} \Rightarrow \boxed{\delta_2 = 46.10^\circ}$$

(assume 0.20% Cast steel untreated for pinion and high grade steel for gear).

(from table 12.7 Pg 234)

$$\sigma_{d1} = 138.3 \text{ MPa}$$

$$\sigma_{d2} = 78.5 \text{ MPa}$$

virtual number of teeth (Z_e):

for bevel gear $Z_e = \frac{Z}{\cos \delta} \rightarrow (\text{eq 12.35 (d) Pg 218})$

$$Z_{e1} = \frac{Z_1}{\cos \delta_1} \Rightarrow \frac{24}{\cos(13.89)} \Rightarrow \boxed{Z_{e1} = 24.72}$$

$$Z_{e2} = \frac{Z_2}{\cos \delta_2} \Rightarrow \frac{72}{\cos(46.10)} \Rightarrow \boxed{Z_{e2} = 103.83}$$

Lewis form factor (y):

$$y = \left(0.124 - \frac{0.684}{Z_{e1}} \right) \rightarrow (\text{eq 12.5 (c) Pg 204})$$

$$y_1 = \left(0.124 - \frac{0.684}{24.72} \right)$$

$$\boxed{y_1 = 0.096 \text{ mm}}$$

$$y_2 = \left(0.124 - \frac{0.684}{103.83} \right)$$

$$\boxed{y_2 = 0.117 \text{ mm}}$$

STEP 1: Identify the weaker member:

Particulars	T_d	γ	$T_d \times \gamma$	Remark
P	138.3	0.096	13.27	
G	78.5	0.117	9.18 ✓	weaker.

Since gear is the weaker member, so the design is based on weaker member,

STEP 2:- Design based on weaker member.

Lewis equation for tangential tooth load.

$$F_t = T_d \times C_v \times b \times \gamma \times m \left[\frac{L-b}{L} \right] \rightarrow (\text{eq 12.37 pg 218})$$

WKT,

$$m_{t2} = \frac{9.55 \times 10^6 \times P \times C_s}{n_2} \rightarrow (\text{eq})$$

$$= \frac{9.55 \times 10^6 \times 9 \times 1.5}{300}$$

$$m_{t2} = 429.75 \times 10^3 \text{ N-mm}$$

$C_s = 1.5$ for
Intermediate
8-10 hrs/day.

$$F_{t2} = \frac{2M_{t2}}{d_2}$$

$$= \frac{2 \times 429.75 \times 10^3}{72 \text{ mm}}$$

$$F_{t2} = \frac{11.95 \times 10^3}{m} \text{ N}$$

$$d = m z$$

$$d_2 = m \times Z_2$$

$$T = F_t \times r$$

$$T = F_t \left(\frac{d}{2} \right)$$

$$F_t = \frac{T}{(d/2)}$$

$$F_t = \frac{2T}{d}$$

$$y = \pi y_2 = \pi(0.117) \Rightarrow \boxed{y = 0.367}$$

NKT, $L = \frac{1}{2} \sqrt{d_1^2 + d_2^2} \longrightarrow (\text{eq 12.33 pg 217}).$

$$L = \frac{1}{2} \sqrt{(mz_1)^2 + (mz_2)^2}$$

$$L = \frac{1}{2} \times m \sqrt{(z_1)^2 + (z_2)^2}$$

$$L = \frac{1}{2} \times m \sqrt{(24)^2 + (72)^2}$$

$$\boxed{L = 37.94 \text{ m}}$$

$$b \leq \frac{L}{3} \longrightarrow (\text{eq 12.36(b) pg 218}).$$

$$b = \frac{L}{3} = \frac{37.94}{3} \Rightarrow \boxed{b = 12.64 \text{ m}}$$

$$m = \frac{F_t}{\tau d \times C_v \times b \times y} \left[\frac{L}{L-b} \right] \longrightarrow (\text{eq 12.37 pg 218})$$

$$m = \frac{11.95 \times 10^3}{m \times 78.5 \times C_v \times 12.64 \text{ m} \times 0.367} \times \left[\frac{3}{2} \right]$$

$$\boxed{m^3 C_v = 49.14}$$

Assume $C_v = 0.5$

$$m^3 (0.5) = 49.14$$

$$\boxed{m = 4.61}$$

(From table 12.2 pg 229) std module $\boxed{m = 5 \text{ mm}}$

$$\frac{L}{(L-b)} = \frac{L}{\left(\frac{3L-L}{3}\right)}$$

$$= \frac{3L}{2L}$$

$$\frac{L}{(L-b)} = \frac{3}{2}$$

Check for $C_v = ?$

$$d_2 = m \times Z_2 \Rightarrow 5 \times 72 \Rightarrow \boxed{d_2 = 360 \text{ mm}}$$

$$V = \frac{\pi d_2 n_2}{60,000} \Rightarrow \frac{\pi \times 360 \times 300}{60,000} \Rightarrow \boxed{V = 5.65 \text{ m/s}}$$

$$C_v = \frac{6.1}{6.1 + V} \longrightarrow (\text{eq 12.38 (b) Pg 219}) \text{ for generated teeth}$$

$$C_v = \frac{6.1}{6.1 + 5.65} \Rightarrow \boxed{C_v = 0.519}$$

$$(m^3 C_v)_{\text{new}} = (5^3 \times 0.519) \Rightarrow \boxed{(m^3 C_v)_{\text{new}} = 64.87}$$

Since $(m^3 C_v)_{\text{new}} > (m^3 C_v)_{\text{module}}$, the design is safe. std $m \pm 5 \text{ mm}$

$$b = 12.64 \text{ m} \Rightarrow 12.64 \times 5 \Rightarrow \boxed{b = 63.2 \text{ mm}}$$

STEP 3: Dimensions:

(from table 12.3 Pg 229)

$$\text{Addendum } (h_a) = m = \underline{\underline{5 \text{ mm}}}$$

$$\text{dedendum } (h_f) = 1.25 \times 5 = \underline{\underline{6.25}}$$

$$\text{Tooth thickness } (t) = 1.5708 \times 5 = \underline{\underline{7.854}}$$

$$\text{working depth} = 2 \times 5 = \underline{\underline{10}}$$

$$\text{whole depth} = 2.25 \times 5 = \underline{\underline{11.25}}$$

$$\text{clearance} = 0.25 \times 5 = \underline{\underline{1.25}}$$

STEP 4: check for dynamic load (F_d):

$$F_d = F_t +$$

$$F_d = F_{t2} + \frac{K_{av} (C_b + F_{t2})}{K_{av} + \sqrt{C_b + F_{t2}}} \rightarrow (\text{eq 12.40 Pg 219})$$

$$F_d = 2390 + \frac{20.67 \times 5.65 (209.96 \times 63.2 + 2390)}{20.67 \times 5.65 + \sqrt{209.96 \times 63.2 + 2390}}$$

$$F_{t2} = \frac{11.95 \times 10^3}{5}$$

$$F_{t2} = 2390 \text{ N}$$

$$F_d = 9949.41 \text{ N}$$

(from table 12.13 Pg 237)

for carefully cut gear ($e_{max} = 0.0277$).

(from table 12.12 Pg 236)

$$C = 0.0277$$

$$0.02 = 151.6$$

$$0.0277 = C$$

$$(0.02 \times C) = (0.0277 \times 151.6)$$

$$C = 209.96$$

STEP 5: wear load (F_w):

$$F_w \geq F_d$$

$$F_w = \frac{d \cdot b \cdot \sigma_c \cdot K}{\cos \delta_1} = F_d$$

$$F_w = \frac{120 \times 63.2 \times 1.615 \times K}{\cos (13.89)} = 9949.41$$

$$\sigma_c = \frac{2Z_{c2}}{(Z_{c2} + Z_{c1})}$$

$$= \frac{2 \times (103.83)}{(103.83 + 24.7)}$$

$$\sigma_c = 1.615$$

$$K \approx 0.79 \leq 0.80$$

For CS and CI $\phi = 14.5^\circ$ involute and $K \approx 0.89$
(from table 12.16 pg 239)

$$\begin{aligned} \text{For pinion CS BHN} &= 250 \\ \text{For gear CI BHN} &= 180 \end{aligned}$$

2. Design a pair of Bevel gear to transmit 25 kW from a shaft rotating @ 1200 rpm to another shaft to rotate @ 500 rpm.

Given data :-

$$\begin{aligned} P &= 25 \text{ kW} \\ n_1 &= 1200 \text{ rpm} \\ n_2 &= 500 \text{ rpm} \end{aligned}$$

Solution.

$$i = \frac{n_1}{n_2} = \frac{1200}{500} \Rightarrow i = 2.4$$

Assume min no of teeth on pinion
 $Z_1 = 20$ teeth.

$$i = \frac{Z_2}{Z_1} \Rightarrow 2.4 = \frac{Z_2}{20} \Rightarrow Z_2 = 48 \text{ teeth}$$

Assume both pinion and gear are made up of same material
(from table 12.7 pg 234) i.e. $\sigma_{d1} = \sigma_{d2}$.

$$\sigma_{d1} = 138.3 \text{ MPa}$$

$$\sigma_{d2} = 138.3 \text{ MPa}$$

If they not mention
assume always $\phi = 20^\circ \text{ FDI}$

Assume $\phi = 20^\circ \text{ FDI}$

Assume pitch cone angle $\theta = 90^\circ$ right angle.

If they not given θ
assume always $(\theta = 90^\circ$
right angle)

$$\tan \delta_1 = \frac{Z_1}{Z_2} \longrightarrow (\text{eq 12.32 (a) pg 217})$$

$$\delta_1 = \tan^{-1} \left[\frac{Z_1}{Z_2} \right]$$

$$\delta_1 = \tan^{-1} \left[\frac{20}{48} \right] \Rightarrow \boxed{\delta_1 = 22.61^\circ}$$

$$\tan \delta_2 = \frac{Z_2}{Z_1} \longrightarrow (\text{eq 12.32 (b) pg 217})$$

$$\delta_2 = \tan^{-1} \left[\frac{48}{20} \right] \Rightarrow \boxed{\delta_2 = 67.38^\circ}$$

Virtual Number of teeth (Z_e) :-

$$Z_e = \frac{Z_1}{\cos \delta_1} \longrightarrow (\text{eq 12.35 (d) pg 218}).$$

$$Z_{e1} = \frac{20}{\cos(22.61)} \Rightarrow \boxed{Z_{e1} = 21.66}$$

$$Z_{e2} = \frac{48}{\cos(67.38)} \Rightarrow \boxed{Z_{e2} = 124.79}$$

Lewis form factor (y) :-

$$y = \left(0.154 - \frac{0.912}{Z_e} \right) \longrightarrow (\text{eq 12.5 (d) pg 204})$$

$$= \left(0.154 - \frac{0.912}{21.66} \right)$$

$$\boxed{y_1 = 0.111}$$

$$y_2 = \left(0.154 - \frac{0.912}{124.79} \right)$$

$$y_2 = 0.146 \text{ mm}$$

STEP 1 :- Identify the weaker member.

Since the pinion and gear made up of same material so pinion is the weaker member. The design is based on weaker member.

STEP 2 :- Design is based on weaker member:

3. Design a pair of metric bevel gear to transmit 7.5 kW @ 1000 rpm. The teeth are 20° stub tooth involute system.

Given data :-

(metric bevel gear)

$$P = 7.5 \text{ kW}$$

$$N_1 = 1000 \text{ rpm}$$

$$\phi = 20^\circ \text{ STI}$$

NOTE :- (For metric bevel gear,
speed ratio $i = 1$, $N_1 = N_2$ and
 $Z_1 = Z_2$, $d_1 = d_2$)

Assume $Z_1 = 20$ teeth for pinion
then $Z_2 = 20$ teeth, $N_2 = 1000 \text{ rpm}$.

Assume for metric bevel gear $\theta = 90^\circ \rightarrow$ right angle.

For pitch cone angle $\delta_1 = 45^\circ$ and $\delta_2 = 45^\circ$

$\delta_1 = \delta_2$ should be
equal to pitch
cone angle.

Assume p & G are made up of 0.20% C
Cast steel untreated.

(from table 12.2 pg 234)

$$\sigma_{d1} = \sigma_{d2}$$

$$\sigma_{d1} = 138.3 \text{ MPa}$$

$$\sigma_{d2} = 138.3 \text{ MPa}$$

virtual number of teeth (Z_e):-

$$Z_e = \frac{Z_1}{\cos \delta_1} \rightarrow \text{(eq 12.35 (d) Pg 218)}$$

$$Z_{e1} = \frac{20}{\cos(45)} \Rightarrow Z_{e1} = 28.28$$

$$Z_{e2} = \frac{20}{\cos(45)} \Rightarrow Z_{e2} = 28.28$$

Lewis form factor (y):

$$y = \left(0.175 - \frac{0.95}{Z_1} \right) \rightarrow (\text{eq 12.5 (c) Pg 204})$$

$$y = \left(0.175 - \frac{0.95}{28.28} \right)$$

$$\boxed{y_1 = 0.141} \quad \boxed{y_2 = 0.141 \text{ mm}}$$

STEP 1 :- Identify the weaker member.

Since pinion and gear are made up of same material so the pinion is the weaker member. Then the design is based on weaker member.

STEP 2 :- Design is based on weaker member.

$$M_{t1} = \frac{9.55 \times 10^6 \times P \times C_s}{n_1} \rightarrow (\text{eq.})$$

$$\frac{9.55 \times 10^6 \times 7.5 \times 1.5}{1000}$$

$$\boxed{M_{t1} = 107.43 \times 10^3 \text{ N-mm}}$$

$$F_{t1} = \frac{2M_{t1}}{m \times Z_1} \Rightarrow \frac{2 \times 107.43 \times 10^3}{m \times 20}$$

$$\begin{aligned} d &= m \times Z_2 \\ d_1 &= m \times Z_1 \end{aligned}$$

$$\boxed{F_{t1} = \frac{107.43 \times 10^3}{m}}$$

$$Y = \pi y_2 = \pi \times (0.141) \Rightarrow \boxed{Y = 0.44 \text{ mm}}$$

face width $(b) = ?$

$$L = \frac{1}{2} \sqrt{d_1^2 + d_3^2} \rightarrow (\text{eq 12.33 pg 217})$$

$$L = \frac{1}{2} \times m \sqrt{(20)^2 + (20)^2}$$

$$L = 14.14 \text{ m}$$

$$\frac{L}{(L-b)} = \frac{L}{\left(\frac{3L-L}{3}\right)} \Rightarrow \frac{34}{24} \Rightarrow \boxed{\frac{L}{(L-b)} = \frac{3}{2}}$$

$$b \leq \frac{L}{3} \rightarrow (\text{eq 12.36(b) pg 218})$$

$$b = \frac{L}{3} = \frac{14.14 \text{ m}}{3} \Rightarrow \boxed{b = 4.71 \text{ m}}$$

$$m = \frac{F_{t1}}{\pi d_1 \psi \times b \times y} \left[\frac{L}{L-b} \right] \rightarrow (\text{eq 12.37 pg 218})$$

$$m = \frac{107.48 \times 10^2}{m \times 138.3 \times \psi \times 4.71 \text{ m} \times 0.44} \left[\frac{3}{2} \right]$$

$$\boxed{m^3 \psi = 56.22}$$

$$(m^3(0.5)) = 56.22 \Rightarrow \boxed{m = 4.82}$$

(from table 12.12 pg 229) standard module $\boxed{m = 5 \text{ mm}}$

$$d_1 = m \times Z_1 = 5 \times 20 \Rightarrow \boxed{d_1 = 100 \text{ mm}}$$

$$v = \frac{\pi d_1 n_1}{60,000} = \frac{\pi \times 100 \times 1000}{60,000} \Rightarrow \boxed{v = 5.23 \text{ m/s}}$$

$$C_v = \frac{6.1}{6.1 + v} \rightarrow (\text{eq 12.38 (b) pg 219}).$$

$$= \frac{6.1}{6.1 + 5.23}$$

$$C_v = 0.538$$

$$(m^3 C_v)_{\text{new}} = (5^3 \times 0.538) \Rightarrow (m^3 C_v)_{\text{new}} = 66.25$$

Since $(m^3 C_v)_{\text{new}} > (m^3 C_v)_{\text{req}}$: so the design is safe.

STEP 4: check for dynamic load (F_d):

$$F_d = F_{t1} + \frac{K_3 V (C_b + F_t)}{K_3 V + \sqrt{C_b + F_t}} \rightarrow (\text{eq$$

(from table 12.13 pg 237)

For metric bevel gear ($e = 0.0277$)

(from table 12.12 pg 236)

$$e = 0.0277$$

$$0.02 = 237.3$$

$$0.0277 = G$$

$$(0.02 \times C) = (0.0277 \times 237.3)$$

$$C = 328.66$$

$$F_{t1} = \frac{107.43 \times 10^2}{m}$$

$$= \frac{107.43 \times 10^2}{5}$$

$$F_{t1} =$$

$$F_d = + \frac{20.67 \times 5.23 (328.66 \times 4.71 + 20.67 \times 5.23 + \sqrt{328.66 \times 4.71})}{20.67 \times 5.23 + \sqrt{328.66 \times 4.71} +}$$

$$F_d = 5.18 \times 10^3$$

STEPS : Wear load (F_w) :

$$F_w \geq F_d$$

$$F_w = \frac{d_1 b Q_c K}{\cos^2 \beta} \geq 5.18 \times 10^3$$

$$F_w = \frac{100 \times 23.55 \times 1 \times K}{\cos^2(45)} \geq 5.18 \times 10^3$$

$$K \geq 1.099$$

(from table 12.16 pg 239) Case steel.

$$\begin{aligned} \text{For pinion steel, BHN} &= 450 \\ \text{For gear steel BHN} &= 350. \end{aligned}$$

$$b = 4.71 \text{ m}$$

$$b = 4.71 \times 5$$

$$b = 23.55$$

$$\begin{aligned} Q_c &= \frac{2Z_{c2}}{(Z_{c2} + Z_{c1})} \\ &= \frac{2 \times 28.28}{(28.28 + 28.28)} \end{aligned}$$

$$Q_c = 1.$$

4. A pair of straight tooth @ right angle bevel gear is to transmit 15KW. @ 1250rpm. of the pinion the diameter of the pinion is 120mm @ the velocity ratio is 3.5. the teeth are 14.5° involute. Determine. The module, face width. from standard point of dynamic load and wear load. The allowable static stress for the pinion is 343.34 MPa and that of a gear is 191.295 MPa.

Given data :-

$$P = 15 \text{ kW}$$

$$N_1 = 1250 \text{ rpm}$$

$$d_1 = 120 \text{ mm}$$

$$i = 3.5 \text{ m/s}$$

$$\phi = 14.5$$

$$m = ?$$

$$b = ?$$

$$\sigma_{d1} = 343.34 \text{ MPa}$$

$$\sigma_{d2} = 191.295 \text{ MPa}$$

$$\theta = 90^\circ \text{ (right angle)}$$

Solution.

$$i = \frac{N_1}{N_2} \Rightarrow 3.5 = \frac{1250}{N_2} \Rightarrow \boxed{N_2 = 357.14 \text{ rpm}}$$

$$i = \frac{d_2}{d_1} \Rightarrow 3.5 = \frac{d_2}{120} \Rightarrow \boxed{d_2 = 420 \text{ mm}}$$

Assume temporary $\boxed{Z_1 = 20 \text{ teeth}}$

$$i = \frac{Z_2}{Z_1} \Rightarrow 3.5 = \frac{Z_2}{20}$$

$$\boxed{Z_2 = 70 \text{ teeth}}$$

If they given dia
we can't assume
teeth
we assume only
temporarily

Pitch cone angle (δ):

$$\delta_1 = \tan^{-1} \left[\frac{d_1}{d_2} \right] \longrightarrow (\text{eq 12.32 (a) pg 217})$$

$$\delta_1 = \tan^{-1} \left[\frac{120}{420} \right]$$

$$\boxed{\delta_1 = 15.94}$$

$$\delta_2 = \tan^{-1} \left[\frac{d_2}{d_1} \right] \longrightarrow (\text{eq 12.32 (b) pg 217})$$

$$\delta_2 = \tan^{-1} \left[\frac{420}{120} \right]$$

$$\boxed{\delta_2 = 74.05}$$

virtual number of teeth (z_c):

$$z_c = \frac{z}{\cos \delta} \rightarrow (\text{eq 12.35 (d) Pg 212})$$

$$z_{c1} = \frac{z_1}{\cos \delta_1} \Rightarrow \frac{20}{\cos(15.94)} \Rightarrow \boxed{z_{c1} = 20.79}$$

$$z_{c2} = \frac{z_2}{\cos \delta_2} \Rightarrow \frac{70}{\cos(74.05)} \Rightarrow \boxed{z_{c2} = 254.73}$$

Lewis form factor (y):

$$y = \left(0.124 - \frac{0.684}{z_c} \right) \rightarrow (\text{eq 12.5 (c) Pg 204}).$$

$$y_1 = \left(0.124 - \frac{0.684}{20.79} \right) \Rightarrow \boxed{y_1 = 0.091 \text{ mm}}$$

$$y_2 = \left(0.124 - \frac{0.684}{254.73} \right) \Rightarrow \boxed{y_2 = 0.121 \text{ mm}}$$

STEP 1 :- Identify the weaker member.

Particular	σ_d	y	$\sigma_d \times y$	Remarks.
PINION	343.34	0.091	31.24	
GEAR.	191.295	0.121	23.14	weaker.

Since gear is the weaker member, so the design is based on gear.

STEP 2 :- Design is based on weaker member:-

$$F_t = V_d \times C_v \times b \times \gamma \times M \left[\frac{L-b}{L} \right] \rightarrow (eq)$$

$$F_t = \frac{1000 \times P \times G_s}{V} = \frac{1000 \times 15 \times 1.5}{7.8} \Rightarrow \boxed{F_t = 2.88 \times 10^3 N}$$

$$V = \frac{\pi d_2 n_2}{60,000} = \frac{\pi \times 420 \times 357.14}{60,000} \Rightarrow \boxed{V = 7.8 m/s}$$

$$C_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 7.8} \Rightarrow \boxed{C_v = 0.438}$$

$$L = \frac{1}{2} \sqrt{d_1^2 + d_2^2} = \frac{1}{2} \sqrt{120^2 + 420^2} \Rightarrow \boxed{L = 218.40 m}$$

$$b = \frac{L}{3} \Rightarrow \frac{218.40 m}{3} \Rightarrow \boxed{b = 72.8 m}$$

$$Z_{c2} = \frac{Z_2}{\cos \delta_2} = \frac{d_2}{m \times \cos \delta_2} = \frac{420}{m \times \cos(74.05)} \Rightarrow \boxed{Z_{c2} = \frac{1.52 \times 10^3}{m}}$$

$$y = \pi \times y_2 \Rightarrow \left(0.124 - \frac{0.684 m}{1.5 \times 10^3} \right) \Rightarrow \boxed{y_2 = 0.124 - 4.56 \times 10^{-4} m}$$

$$\left(\frac{L}{L-b} \right) = \frac{L}{\left(\frac{3L-L}{3} \right)} = \frac{34}{24} \Rightarrow \boxed{\left(\frac{L}{L-b} \right) = \frac{3}{2}}$$

$$2.88 \times 10^3 = 191,295 \times 0.438 \times 72.8 \times \pi \left(0.124 - 4.56 \times 10^{-4} m \right)$$

$$\times m \left[\frac{3}{2} \right]$$

$$2.88 \times 10^3 = 12.74 \times 10^3 m \left(0.124 - 4.56 \times 10^{-4} m \right)$$

$$2.88 \times 10^3 = 1.58 \times 10^3 m - 5.73 m^2$$

$$5.73 \times 10^4 - 1.58 \times 10^3 m + 2.88 \times 10^3$$

$$m = 273.91 \text{ and } 1.8$$

(from table 12.12 pg 229) standard module $m = 2 \text{ mm}$

WKT $d = m \times Z \Rightarrow Z_1 = \frac{d_1}{m} = \frac{120}{2} \Rightarrow Z_1 = 60 \text{ teeth}$

$$Z_2 = \frac{d_2}{m} = \frac{420}{2} \Rightarrow Z_2 = 210 \text{ teeth}$$

STEP 3 :- check for dynamic load (SS) :-

$$F_d = F_{t2} + \frac{K_3 V (C_b + F_t)}{K_3 V + \sqrt{C_b + F_t}} \longrightarrow (\text{eq 12.40 pg 219})$$

To find dynamic factor 'C'

(from table 12.14 pg 237).

$$e = 0.05$$

(From fig. 12.12 pg 236) for $\phi = 45^\circ$

$$e = 0.05$$

$$0.04 = 441.8$$

$$0.05 = C$$

$$(0.04 \times C) = (0.05 \times 441.8)$$

$$C = 551.62$$

if they not given material to find error, we can go through velocity and then find error.

$$F_d = 2.88 \times 10^3 + \frac{20.67 \times 7.8 (551.62 \times 72.8 + 2.88 \times 10^3)}{20.67 \times 7.8 + \sqrt{551.62 \times 72.8 + 2.88 \times 10^3}}$$

$$F_d = 21.700 \times 10^3 \text{ N}$$

STEP 5: wear load (F_w):

$$F_w \geq F_d$$

$$F_w = \frac{d_1 b Q_c K}{\cos^2 \beta} \geq F_d$$

$$F_w = \frac{120 \times}{\cos^2 \beta}$$

$$K = 1.29$$

(from table 12.16 pg 239) for $K = 1.29$

$$\begin{aligned} \text{For pinion BHN} &= 350 \\ \text{For gear BHN} &= 350 \end{aligned}$$

$$Q_c = \frac{2Z_{c2}}{Z_{c2} + Z_{c1}}$$

$$= 2 \times$$

$$Q_c =$$

5. A pair of Bevel gear is made of gray CI. FG-200 having young's modulus of 114 GPa. The surface endurance strength is 90 MPa. The number of teeth on Pinion and gear are 30 and 40 respectively. The Pinion rotates @ 800 rpm and gear rotates @ 600 rpm. The module is face width are 60 and 50 mm respectively. The pressure angle is 20°. Determine the wear strength of the tooth. The shafts are @ 70° to each other. What is the power capacity of the gear pair.

Given data :-

(gray cast iron $FG = 200$).

$$E = 114 \text{ GPa} = 114 \times 10^3 \text{ MPa}$$

$$\tau_{\text{en}} = 90 \text{ MPa}$$

$$Z_1 = 30 \text{ teeth}$$

$$Z_2 = 40 \text{ teeth}$$

$$r_1 = 800 \text{ mm}$$

$$r_2 = 600 \text{ mm}$$

$$m = 6 \text{ mm}$$

$$b = 50 \text{ mm}$$

$$\phi = 20^\circ$$

$$p = ?$$

$$F_w = ?$$

$$\theta = 70^\circ$$

solution.

WKT, (for bevel gear).

$$F_w = \frac{d_1 b Q_c K}{\cos \delta_1} \rightarrow (\text{eq 12.41 pg 219})$$

$$d = m \times Z_1 = 6 \times 30 \Rightarrow \boxed{d_1 = 180 \text{ mm}}$$

For add angle.

Pitch cone angle of pinion.

$$\tan \delta_1 = \frac{\sin \theta}{\frac{Z_2}{Z_1} + \cos \theta} \rightarrow (\text{eq 12.2 (a) pg 215})$$

$$\delta_1 = \tan^{-1} \left[\frac{\sin 70}{\frac{40}{30} + \cos 70} \right] \Rightarrow \boxed{\delta_1 = 29.28^\circ}$$

$$\delta_2 = \tan^{-1} \left[\frac{\sin 70}{\frac{30}{40} + \cos 70} \right] \Rightarrow \boxed{\delta_2 = 40.71^\circ}$$

virtual number of teeth (Z_c)

$$Z_c = \frac{Z_1}{\cos \delta} \rightarrow (\text{eq 12.35 (d) pg 218})$$

$$Z_{c1} = \frac{30}{\cos(29.28)} \Rightarrow \boxed{Z_{c1} = 34.39}$$

$$Z_{c2} = \frac{40}{\cos(40.71)} \Rightarrow \boxed{Z_{c2} = 52.76}$$

$$\delta_c = \frac{2Z_{c2}}{Z_{c1} + Z_{c2}} \Rightarrow \frac{2 \times 52.76}{34.39 + 52.76} \Rightarrow \boxed{\delta_c = 1.21}$$

$$K = \frac{\tau_{\text{ten}}^2 \sin^2}{1.40} \left[\frac{1}{E_1} + \frac{1}{E_2} \right] \rightarrow (\text{eq 12.2 (c) pg 214})$$

$$K = \frac{90^2 \times \sin^2(20)}{1.40} \left[\frac{1}{114 \times 10^3} + \frac{1}{114 \times 10^3} \right] \Rightarrow \boxed{K = 0.034}$$

$$F_w = \frac{d_{ib} \delta_c K}{\cos \delta_1} = \frac{180 \times 50 \times 1.21 \times 0.034}{\cos(29.28)} \Rightarrow \boxed{F_w = 424.49 \text{ N}}$$

$$v = \frac{\pi d_1 N_1}{60,000} = \frac{\pi \times 180 \times 300}{60,000} \Rightarrow \boxed{v = 7.53 \text{ m/s}}$$

(from table 12.7 pg 234) for high grade C1 heat treated.

$$\tau_{d1} = 78.5 \text{ MPa}$$

$$C_v = \frac{6.1}{6.1 + v} \rightarrow (12.38 (b) \text{ pg 219})$$

$$C_v = \frac{6.1}{6.1 + 7.53} \Rightarrow \frac{6.1}{6.1 + 7.53} \Rightarrow \boxed{C_v = 0.44}$$

$$y = \left(0.154 - \frac{0.912}{34.39} \right) \Rightarrow \boxed{y_1 = 0.127 \text{ mm}}$$

$$\frac{L-b}{L} = \frac{2}{3}$$

$$F_t = \tau_{d1} \times C_v \times b \times y \times m \left(\frac{L-b}{L} \right)$$

$$= 78.5 \times 0.44 \times 50 \times \pi (0.127) \times 6 \left[\frac{2}{3} \right]$$

$$\boxed{F_t = 2756.17 \text{ N}}$$

Power capacity (P) = ?

$$F_t = \frac{1000 \times P \times C_s}{V} \Rightarrow 2756.17 = \frac{1000 \times P \times 1.5}{7.53} \Rightarrow \boxed{P = 13.82 \text{ kW}}$$

Worm Gear

14/06/2022
Tuesday

Worm Gear :- Worm Gear are widely used to transmit power @ high velocity ratio between non parallel and non intersecting shafts usually @ right angles to each other

- * The worm gear drive consists of threaded worm in mesh with the gear.
- * The worm can be single, double, triple (or) even more number of starts of threads.

$$i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1} \neq \frac{d_2}{d_1}$$

$$d_2 = m \times Z_2 \quad d_1 \neq m \times Z_1$$

material for threaded worm = hardened steel

material for worm wheel CI = phosphorus.

Problems:-

1. Design a worm gear drive to transmit 18kW from a worm rotating @ 1440rpm. To a worm wheel to rotate @ 40rpm check the heating capacity of the gear and determine the efficiency.

Given data:-

$$\begin{aligned} P &= 18 \text{ kW} \\ n_1 &= 1440 \text{ rpm} \\ n_2 &= 40 \text{ rpm} \\ \eta &=? \\ H_d \text{ \& } H_g &=? \end{aligned}$$

solution:-

$$i = \frac{n_1}{n_2} = \frac{1440}{40} \Rightarrow \boxed{i = 36}$$

(from table 12.28(b) pg 244) for $i = 36$.

No of threads/start on worm gear $\boxed{Z_1 = \text{single}}$

$$i = \frac{Z_2}{Z_1} \Rightarrow 36 = \frac{Z_2}{1} \Rightarrow \boxed{Z_2 = 36 \text{ teeth}}$$

(from table 12.7 pg 234).

Assume material for worm $Vd_1 = \text{steel C30 heat treated.}$
material for worm wheel $Vd_2 = \text{phosphorus.}$

$$\therefore \boxed{Vd_1 = 220.6 \text{ MPa}}$$

$$\boxed{Vd_2 = 82.4 \text{ MPa}}$$

(from table 12.27 pg 244)

for single start thread, the pressure angle $\boxed{\alpha = 14.5^\circ}$

STEP 1:- Identification the member member:-

In worm gear design the worm wheel is the weaker member. so design is based on worm wheel.

STEP 2 :- Design is based on material member :-

Lewis equation for tangential tooth load

$$F_t = \sigma_d \times C_v \times b \times y \times m \rightarrow (\text{eq 12.53 (a) pg 223})$$

$$F_t = \frac{2M_{t2}}{d_2} = \frac{2 \times 6.44 \times 10^6}{36m} \Rightarrow \boxed{F_t = \frac{357.77 \times 10^3}{m}}$$

$$M_{t2} = \frac{9.55 \times 10^6 \times P \times C_s}{n_2} \Rightarrow \frac{9.55 \times 10^6 \times 18 \times 1.5}{40} \Rightarrow \boxed{M_{t2} = 6.44 \times 10^6 \text{ N-mm}}$$

$$y = \pi y_2 : (\text{from table 12.28 (c) pg 244}) \text{ for } d = 14.5^\circ \text{ then } \boxed{y = 0.1}$$

$$(\text{from Table 12.27 pg 244}) \boxed{b = 7.48m + 6.35 \text{ mm}}$$

* don't consider 6.35 value while you calculate m's form. only add 7.48 m

$$m = \frac{F_t}{\sigma_d \times C_v \times b \times y} = \frac{357.77 \times 10^3}{m \times 82.4 \times C_v \times 7.48m \times \pi \times 0.1} \Rightarrow \boxed{m^3 C_v = 1847.67}$$

$$\underline{C_v = 0.9} \Rightarrow (m^3 \times 0.9) = 1848.67 \Rightarrow \boxed{\text{module } m = 12.71}$$

$$(\text{from table 12.2 pg 229}) \text{ std module } \boxed{m = 16 \text{ mm}}$$

For WG C_v value 0.8 to 0.95. we always take 0.95.

$$v = \frac{\pi d_2 n_2}{60,000} = \frac{\pi \times 576 \times 40}{60,000} \Rightarrow \boxed{v = 1.2 \text{ m/s}}$$

$$d = m \times Z_2 = 16 \times 36$$

$$C_v = \frac{6.1}{6.1 + v} \rightarrow (\text{from eq 12.53 (c) pg 223})$$

$$\boxed{d_2 = 576 \text{ mm}}$$

$$C_v = \frac{6.1}{6.1 + 1.2} \Rightarrow \boxed{C_v = 0.83}$$

take C_v value always $\frac{6.1}{6.1 + v}$

$$(m^3 C_v)_{\text{new}} > (16^3 \times 0.83)_{\text{new}} \Rightarrow \boxed{(m^3 C_v)_{\text{new}} = 3399.68}$$

Since $(m^3 C_v)_{\text{new}} > (m^3 C_v)_{\text{req}}$ so the design is safe @ $m = 16 \text{ mm}$

STEP 3 :- Dimensions :-

(from Table 12.27 Pg 244)

$$\text{Face width } (b) = 7.48m + 6.35 \Rightarrow 7.48 \times 16 + 6.35 \Rightarrow \boxed{b = 126.03 \text{ mm}}$$

$$\text{Face length } (L_w) = (14.14 + 0.06321)m \Rightarrow (14.14 + 0.06321) \times 16 \Rightarrow \boxed{L_w = 227.15 \text{ mm}}$$

$$\text{Outside dia} \Rightarrow d_2 + 3.1854 \times m \Rightarrow 576 + 3.1854 \times 16 \Rightarrow \boxed{d_o = 626.96 \text{ mm}}$$

$$\text{Throat dia} \Rightarrow d_2 + 2m \Rightarrow 576 + 2 \times 16 \Rightarrow \boxed{d_t = 608 \text{ mm}}$$

$$\text{Radius of gear face} \Rightarrow 2.772m + 14 \Rightarrow 2.772 \times 16 + 14 \Rightarrow \boxed{r_b = 58.352 \text{ mm}}$$

$$\text{Radius of gear rim} \Rightarrow 6.915m + 14 \Rightarrow 6.915 \times 16 + 14 \Rightarrow \boxed{r_r = 124.64 \text{ mm}}$$

$$\text{Radius of edge} \Rightarrow 0.7858m \Rightarrow 0.7858 \times 16 \Rightarrow \boxed{r_e = 12.5728 \text{ mm}}$$

STEP 4 :- Dynamic load (F_d)

(from eq 12.54 Pg 223)

$$F_d = Vd \times b \times Y_2 m.$$

$$F_d = 82.4 \times 126.03 \times \pi \times 0.1 \times 16$$

$$\boxed{F_d = 52.200 \times 10^3 \text{ N}}$$

STEP 5 :- Wear load (F_w) :-

$$F_w > F_d.$$

$$F_w = d_2 b K \rightarrow (\text{eq 12.62 (a) Pg 227}).$$

To find lead angle (γ).

$$\tan \gamma = \sqrt[3]{n_2/n_1} \rightarrow (\text{eq 12.52 (b) Pg 223}).$$

$$\gamma = \tan^{-1} \sqrt[3]{n_2/n_1}$$

$$\gamma = \tan^{-1} \sqrt[3]{\frac{40}{1440}} \Rightarrow \boxed{\gamma = 16.34^\circ}$$

For hardened steel and phosphorus bronze lead angle $\gamma = 16.24^\circ$
(from table 12.30 pg 246)

Lead stress factor $\boxed{K = 0.687}$

$$F_W = 576 \times 126.03 \times 0.687 \Rightarrow \boxed{F_W = 49.87 \times 10^3 \text{ N}}$$

Since $F_W < F_d$ and design is not safe. for safe design,

WKT, $F_W > F_d$

$$d_2 \times b \times K > 52.86 \times 10^3$$

$$K \geq \frac{52.86 \times 10^3}{576 \times 126.03} \quad \frac{F_d}{d_2 \times b}$$

$$\boxed{K \geq 0.71}$$

$$d_1 = \frac{Z_1 m_2}{\tan \gamma} \rightarrow (\text{eq 12.46 (h) pg 221})$$

$$d_1 = \frac{1 \times 16}{\tan(16.34)} \Rightarrow \boxed{d_1 = 52.86 \text{ mm}}$$

STEP 6:- Amount of heat generated

$$H_g = \frac{\mu F_n v_r}{\cos \gamma} \rightarrow (\text{eq 12.63 (a) pg 227})$$

Assume friction co-eff $\mu = 0.05$

$$F_f = \frac{357.77 \times 10^3}{m} \Rightarrow \frac{357.77 \times 10^3}{16}$$

$$\boxed{F_f = 22.66 \times 10^3 \text{ N}}$$

If they not given
assume the co-eff of
friction $\mu = 0.05$

$$V_r = \frac{\pi d_1 n_1}{6000 \cos \gamma} \Rightarrow \frac{\pi \times 52.86 \times 1440}{6000 \times \cos(16.84)} \Rightarrow \boxed{V_r = 4.96 \text{ m/s}}$$

$$F_n = \frac{F_t}{\cos \alpha \times \cos \gamma} \Rightarrow \frac{22.36 \times 10^3}{\cos(14.5) \times \cos(16.84)} \Rightarrow \boxed{F_n = 24.13 \times 10^3 \text{ N}}$$

$$\dot{Q} = \frac{4 F_n \times V_r}{\cos \gamma} \Rightarrow \frac{0.05 \times 24.13 \times 10^3 \times 4.96}{\cos(16.84)} \Rightarrow \boxed{\dot{Q} = 5.24 \times 10^3 \text{ Watts}}$$

STEP:- Amount of heat dissipated (\dot{Q}):-

$$\dot{Q} = 1000 (\text{kW}) (1 - \eta) \rightarrow (\text{eq 12.63 (b) eq 227}).$$

For bas's formula of η of worm gear.

$$\eta = \frac{\tan \gamma (1 - u \tan \gamma)}{u + \tan \gamma} \rightarrow (\text{eq 12.57 (d) Pg 225})$$

$$= \frac{\tan(16.84) (1 - 0.05 \times \tan(16.84))}{0.05 + \tan(16.84)}$$

$$\boxed{\eta = 0.845\%}$$

$$\dot{Q} = 1000 \times 18 \times (1 - 0.845) \Rightarrow \boxed{\dot{Q} = 2.79 \text{ kW}}$$

9. Design a worm drive for a speed reducer to transmit 30kW @ a worm speed of 600rpm. the required velocity ratio is 25:1. The worm is made of C30 heat treated steel and worm wheel made of phosphorus bronze. the service conditions are intermittent operation with medium shock load also calculate heat dissipation through the device

Given data:

$$P = 30 \text{ kW}$$

$$n_1 = 600 \text{ rpm}$$

$$i = 25:1$$

C30 steel for - worm
phosphorus for - worm wheel
Intermittent operation with
medium shock $S = ?$

Solution:

$$i = \frac{n_1}{n_2} = 25 = \frac{600}{n_2} \Rightarrow \boxed{n_2 = 24 \text{ rpm}}$$

(from table 12.28 (b) pg 244)

for $i = 25$ no of threads on
worm gear $Z_1 = \text{single}$

$$i = \frac{Z_2}{Z_1} \Rightarrow 25 = \frac{Z_2}{1} \Rightarrow \boxed{Z_2 = 25}$$

(from table 12.7 pg 234)

Assume material for worm $\sigma_{d1} = \text{steel, C30 heat treated}$
material for worm wheel $\sigma_{d2} = \text{phosphorus}$

i.e $\boxed{\sigma_{d1} = 220.6 \text{ MPa}}$

$$\boxed{\sigma_{d2} = 82.4 \text{ MPa}}$$

(from table 12.27 pg 244)

For single start thread, the pressure angle $\boxed{\alpha = 14.5}$

STEP 1 :- Identify the weaker member :-

In worm gear design, the worm wheel is the weaker member. so the design is based on worm wheel.

STEP 2 :- Design is based on weaker member.

Derive equation for tangential tooth load.

$$F_t = \sigma_d \times C_v \times b \times \gamma \times m \rightarrow (\text{eq 12.53(a) pg 228})$$

$$F_t = \frac{2M_{t2}}{d_2} = \frac{2 \times 14.92 \times 10^6}{250} \Rightarrow \boxed{F_t = \frac{1.193 \times 10^6}{m}}$$

$$M_{t2} = \frac{9.55 \times 10^6 \times 30 \times 1.25}{24} \Rightarrow \boxed{M_{t2} = 14.92 \times 10^6 \text{ N-mm}}$$

$y = \pi y_2$ (from table 12.28 pg 244) for $d = 14.5^\circ$ then $\boxed{y = 0.1}$
(from table 12.27 pg 244) $\boxed{b = 7.48m + 6.35mm}$

$$m = \frac{F_{t2}}{\pi d \times C_v \times b \times y} = \frac{1.193 \times 10^6}{m \times 82.4 \times C_v \times 7.48m \times \pi \times 0.1} \Rightarrow \boxed{m^3 C_v = 6161.14}$$

$C_v = 0.9 \Rightarrow (m^3(0.9)) = 6161.14 \Rightarrow \boxed{\text{module } m = 18.98}$

(from table 12.2 pg 229) standard module $\boxed{m = 20mm}$

$$v = \frac{\pi d_2 n_2}{60,000} \Rightarrow \frac{\pi \times 500 \times 24}{60,000} \Rightarrow \boxed{v = 0.62 \text{ m/s}}$$

$d = m \times Z_1$
 20×25

$$C_v = \frac{6.1}{6.1 + v} \Rightarrow \frac{6.1}{6.1 + 0.62} \Rightarrow \boxed{C_v = 0.90}$$

$\boxed{d_2 = 500}$

$$(m^3 C_v)_{\text{new}} = (20^3 \times 0.90)_{\text{new}} \Rightarrow \boxed{(m^3 C_v)_{\text{new}} = 7200}$$

Since $(m^3 C_v)_{\text{new}} > (m^3 C_v)_{\text{req}}$ so the design is safe for $m = 20mm$

STEP 3 :- Dimension :-

(from table 12.27 pg 244)

$$b = 7.48m + 6.35 \Rightarrow 7.48 \times 20 + 6.35 \Rightarrow b =$$

$$L_w = (14.14 + 0.06321)m \Rightarrow$$

$$d_o = d_2 + 3.1854 \times m \Rightarrow 500 + 3.1854 \times 20 \Rightarrow d_o =$$

$$d_t = d_2 + 2m \Rightarrow 500 + 2 \times 20 \Rightarrow d_t$$

$$y_b = 2.772m + 14 \Rightarrow 2.772 \times 20 + 14 \Rightarrow y_b$$

$$y_r = 6.915m + 14 \Rightarrow 6.915 \times 20 + 14 \Rightarrow y_r$$

$$y_c = 0.7858m \Rightarrow 0.7858 \times 20 \Rightarrow y_c$$

STEP 4:- Dynamic load (F_d):

(from eq 12.54 pg 223)

$$F_d = V_d \times b \times y \times m$$

$$= 82.4 \times 155.95 \times \pi \times 0.1 \times 20$$

$$F_d = 80.74 \times 10^3 \text{ N}$$

STEP 5:- Wear load:

$$F_w > F_d$$

$$F_w = d_2 b K \rightarrow (\text{eq 12.62 (a) pg 227}).$$

To find lead angle (γ).

$$\gamma = \tan^{-1} \sqrt[3]{n_2/n_1} \rightarrow (\text{eq 12.52 (b) pg 223})$$

$$\gamma = \tan^{-1} \sqrt[3]{\frac{24}{600}}$$

$$\gamma = 18.88$$

For steel C30 heat treated and phosphorus bronze $\gamma = 18.88$
(from table 12.30 pg 246)

$$\text{Lead stress factor } K = 0.687$$

Since $F_w < F_d$ and design is not safe. for safe design

WKT, $F_w > F_d$

$$d_{2bk} > F_d$$

$$d_{2bk} > 80.74 \times 10^3$$

$$K \geq \frac{80.74 \times 10^3}{500 \times 155.95 \times 0.687}$$

$$K \geq 1.03$$

STEP 6:- Amount of heat generated:-

$$H_g = \frac{\mu F_n V_r}{\cos \gamma} \rightarrow \text{(eq 12.63(a) pg 227)}$$

Assume friction Co-efficient is $\mu = 0.05$

$$F_t = \frac{1.193 \times 10^6}{n} = \frac{1.193 \times 10^6}{20} \Rightarrow F_t = 59.65 \times 10^3 \text{ N}$$

$$V_r = \frac{\pi d_1 n_1}{6000 \times \cos \gamma} = \frac{\pi \times 58.48 \times 600}{60,000 \times \cos(18.88)} \Rightarrow V_r = 1.94 \text{ m/s}$$

$$d_1 = \frac{Z_1 \times m}{\tan \gamma} = \frac{1 \times 20}{\tan(18.88)} \Rightarrow d_1 = 58.48$$

$$F_n = \frac{F_t}{\cos \alpha \times \cos \gamma} = \frac{59.65 \times 10^3}{\cos(14.5) \times \cos(18.88)} \Rightarrow F_n = 65.11 \times 10^3 \text{ N}$$

$$H_g = \frac{\mu F_n \times V_r}{\cos \gamma} = \frac{0.05 \times 65.11 \times 10^3 \times 1.94}{\cos(18.88)} \Rightarrow H_g = 6.67 \times 10^3 \text{ watt}$$

STEP 7: Amount of heat dissipated (Q):

$$Q = 1000 \text{ (KW)} (1 - \eta) \rightarrow (\text{eq 12.63 (b) eq 227})$$

for Lewis formula of η of worm gear.

$$\eta = \frac{\tan \phi (1 - u \tan \phi)}{u + \tan \phi} \rightarrow (\text{eq 12.57 (d) pg 225})$$

$$= \frac{\tan (18.86) (1 - 0.05 \times \tan (18.88))}{0.05 + \tan (18.88)} \Rightarrow \boxed{\eta = 0.85}$$

$$Q = 1000 \times 30 \times (1 - 0.857) \Rightarrow \boxed{Q = 4.29 \text{ KW}}$$

3. Complete the design and determine the Input Capacity of worm gear speed reducer unit. which consists of a hardened steel worm and phosphorus bronze gear having 20°SIT . the center distance is to be 200mm. and a transmission ratio is 10, and worm speed is 2000rpm.

Given data:

worm - hardened steel
wheel - phosphorus bronze

$$\alpha = 20^\circ \text{SIT}$$

$$a = 200 \text{ mm}$$

$$i = 10$$

$$N_1 = 2000$$

Complete design = ?

Input capacity power = ?

Solution:-

$$i = \frac{n_1}{n_2} = 10 = \frac{2000}{n_2} \Rightarrow \boxed{n_2 = 200 \text{ rpm}}$$

(from table 12.28 (b) pg 244)

for $i = 10$, No. of threads on worm gear $Z_1 = \text{triple (3)}$.

$$i = \frac{Z_2}{Z_1} = 10 = \frac{Z_2}{3} \Rightarrow \boxed{Z_2 = 30 \text{ teeth}}$$

(From table 12.7 pg 234)

Assume material for worm = hardened steel

material for worm wheel = phosphorus bronze.

$$\tau_{d1} = 220.6 \text{ MPa}$$

$$\tau_{d2} = 82.4 \text{ MPa}$$

$$\text{WKT } a = \frac{d_1 + d_2}{2} = 200 = \frac{70.35 + d_2}{2} \Rightarrow d_2 = 329.65 \text{ mm}$$

According to AGMA, (American gear manufacturing association)

$$d_1 = \frac{a^{0.875}}{1.466} \rightarrow (\text{eq 12.51 (a) pg 223})$$

$$d_1 = \frac{200^{0.875}}{1.466}$$

$$d_1 = 70.35 \text{ mm}$$

$$\text{Lead angle } \tan \psi = \sqrt[3]{n_2/n_1} \rightarrow (\text{eq 12.52 (b) pg 223})$$

$$\psi = \tan^{-1} \sqrt[3]{\frac{200}{2000}} \Rightarrow \psi = 24.83^\circ$$

STEP 1: Identify the weaker member.

In worm gear design, the worm wheel is the weaker member so the design based on worm wheel.

STEP 2: Design is based on weaker member.

Lewis Equation for tangential tooth load.

$$F_t = T_d \times Q \times b \times Y \times m \rightarrow (\text{eq 12.53(a) pg 223})$$

To find power (P) in terms of SI unit (a) kW.

$$P(\text{kW}) = 0.02905 \times \frac{Q^{1.7}}{i' + 5} \rightarrow (\text{eq 12.68(a) pg 228})$$

$$= 0.02905 \times \frac{200^{1.7}}{10 + 5}$$

$$\boxed{P = 15.80 \text{ kW}}$$

$$i' = \frac{n_1}{n_2} = \frac{10}{2000} = 0.005$$

$$\boxed{i' = 10}$$

$$M_{t2} = \frac{9.55 \times 10^6 \times 15.80 \times 1.5}{200} \Rightarrow \boxed{M_{t2} = 1.131 \times 10^6 \text{ N-mm}}$$

$$F_t = \frac{2M_{t2}}{d_2} = \frac{2 \times 1.131 \times 10^6}{329.65} \Rightarrow \boxed{F_t = 6861.82 \text{ N}}$$

$$Y = \pi Y_2 \text{ (from table 12.28 pg 244) for } d = 20^\circ: \text{ then } \boxed{Y = 0.125}$$

$$V = \frac{\pi d_2 n_2}{60,000} = \frac{\pi \times 329.65 \times 200}{60,000} \Rightarrow \boxed{V = 3.45 \text{ m/s}}$$

$$Q = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 3.45} \Rightarrow \boxed{Q = 0.638}$$

$$\text{(from table 12.27 pg 244)} \quad b = 6.758m + 5.08mm.$$

$$6861.82 = 82.4 \times 0.638 \times 6.758m \times \pi \times 0.125 \times m$$

$$\boxed{m = 7.08 \text{ mm}}$$

$$\text{(from table 12.2 pg 229) standard module } \boxed{m = 8 \text{ mm}}$$

STEP 3:- Dimensions:-

$$b = 6.758m + 5.08 = 6.758 \times 8 + 5.08 \Rightarrow b = 59.144mm$$

$$LW = (14.14 + 0.063Z_1)m = (14.14 + 0.063 \times 3)8 \Rightarrow LW =$$

$$d_o = d_2 + 2.7982m = 329.65 + 2.7982 \times 8 \Rightarrow d_o =$$

$$d_t = d_2 + 1.7978m = 329.65 + 1.7978 \times 8 \Rightarrow d_t =$$

$$r_b = 2.873m + 14 = 2.873 \times 8 + 14 \Rightarrow r_b =$$

$$r_r = 6.6m + 14 = 6.6 \times 8 + 14 \Rightarrow r_r =$$

$$r_c = 0.7858m = 0.7858 \times 8 \Rightarrow r_c =$$

STEP 4:- Dynamic load.

$$F_d = Vd \times b \times Y \times m \Rightarrow 82.4 \times 59.144 \times \pi \times 0.125 \times 8$$

$$\boxed{F_d = 15.30 \times 10^3 N}$$

For hardened steel and phosphorus bronze $\gamma = 24.88^\circ$
(from table 12.30 Pg 246)

$$\text{Load stress factor } \boxed{K = 0.687}$$

$$F_w = d_2 b K = 329.65 \times 59.144 \times 0.687 \Rightarrow \boxed{F_w = 13.39 \times 10^3 N}$$

Design is not safe, so for the safe design,

$$F_w > F_d$$

$$d_2 b K > 15.30 \times 10^3$$

$$K \geq \frac{15.30 \times 10^3}{329.65 \times 59.144} \Rightarrow \boxed{K \geq 0.78}$$

STEP 6: Amount of heat generated.

$$H_g = \frac{\mu F_n V_r}{\cos \gamma} \rightarrow (\text{eq 12.63(a) Pg 227}).$$

$$V_r = \frac{\pi d_i n_i}{60.000 \cos \gamma} = \frac{\pi \times 70.35 \times 2000}{60.000 \times \cos(24.83)} \Rightarrow \boxed{V_r = 8.12 \text{ m/s}}$$

$$F_n = \frac{F_t}{\cos \alpha \times \cos \gamma} = \frac{6861.82}{\cos(20) \times \cos(24.83)} \Rightarrow \boxed{F_n = 8.04 \times 10^3 \text{ N}}$$

$$H_g = \frac{0.05 \times 8.04 \times 10^3 \times 8.12}{\cos(24.83)} \Rightarrow \boxed{H_g = 3.59 \times 10^3 \text{ watts}}$$

STEP 7: Amount of heat dissipated (H_d).

$$\dot{Q} = 1000 [\text{KW}] (1 - \eta) \rightarrow (\text{eq 12.63 (b) eq 227}).$$

$$\eta = \frac{\tan \gamma (1 - \mu \tan \gamma)}{\mu + \tan \gamma} \rightarrow (\text{eq 12.57 (d) Pg 225})$$

$$= \frac{\tan(24.83) (1 - 0.05 \times \tan(24.83))}{0.05 + \tan(24.83)}$$

$$\boxed{\eta = 0.88}$$

$$\dot{Q} = 1000 \times 3.59 \times (1 - 0.88) \Rightarrow \boxed{\dot{Q} = 1.89 \text{ KW}}$$

4. The following data refers to a worm gear and worm gear drive center distance equal 200mm. Pitch circle diameter of worm = 80mm. No of starts = 4. Addendum module = 8mm. Transmission ratio = 20, the worm gear is made up of phosphorus bronze with an allowable bending stress of 55MPa. The worm is made of hardened ground steel with allowable stress is 220.6MPa. Tooth forming is 20° FDI. Determine the following,

1. No of teeth on worm gear.
2. Lead angle
3. face width of worm gear.

To transmit 15KW of power @ 1750rpm. of the worm based on the beam strength of worm gears.

Given data:

$$a = 200 \text{ mm}$$

$$d_1 = 80 \text{ mm}$$

$$z_1 = 4$$

$$m = 8 \text{ mm}$$

$$i = 20$$

$$P = 15 \text{ kW}$$

$$n_1 = 1750 \text{ rpm}$$

$$\gamma = ?$$

$$\sigma_{d1} = 220.6 \text{ MPa}$$

$$\sigma_{d2} = 55 \text{ MPa}$$

$$\phi = 20^\circ \text{ FDI}$$

$$z_2 = ? \quad b = ?$$

Solution:-

$$i = \frac{n_1}{n_2} \Rightarrow 20 = \frac{1750}{n_2} \Rightarrow \boxed{n_2 = 87.5 \text{ rpm}}$$

$$i = \frac{z_1}{z_2} \Rightarrow 20 = \frac{4}{z_2} \Rightarrow \boxed{z_2 = 80 \text{ mm}}$$

STEP 1 :- Identify the weaker member.

In worm gear design, the worm wheel is the weaker member. So the designed is based on weaker member.

STEP 2: Design is based on master member.

$$F_t = \pi d \times C_v \times b \times \gamma \times m \rightarrow (\text{eq 12.53(a) pg 223})$$

$$M_2 = \frac{9.55 \times 10^6 \times P \times C_s}{n_2} = \frac{9.55 \times 10^6 \times 15 \times 1.5}{87.5} \Rightarrow \boxed{M_{t2} = 2.45 \times 10^6 \text{ N-m}}$$

$$F_{t2} = \frac{2M_{t2}}{d_2} = \frac{2 \times 2.45 \times 10^6}{5} \Rightarrow \boxed{F_{t2} = 15.3125 \text{ N}}$$

$$a = \frac{d_1 + d_2}{2} \Rightarrow 200 = \frac{80 + d_2}{2} \Rightarrow \boxed{d_2 = 5 \text{ mm}}$$

$$\tan \nu = \sqrt[3]{\frac{n_2}{n_1}} \Rightarrow \nu = \tan^{-1} \sqrt[3]{\frac{87.5}{1750}} \Rightarrow \boxed{\nu = 20.22}$$

$$\gamma = \pi \gamma_2 \Rightarrow (\text{table 12.28 pg 244}) \alpha = 20^\circ \text{ then } \boxed{\gamma_2 = 0.125}$$

$$C_v = \frac{6.1}{6.1 + v} \Rightarrow \frac{6.1}{6.1 + 1.46} \Rightarrow \boxed{C_v = 0.8068}$$

$$v = \frac{\pi d_2 n_2}{60,000} \Rightarrow \frac{\pi \times 320 \times 87.5}{60,000} \Rightarrow \boxed{v = 1.46 \text{ m/s}}$$

$$d_2 = m \times Z_2$$
$$8 \times 80$$

$$d_2 = 640/2$$

$$\boxed{d_2 = 320}$$

$$F_t = \pi d \times C_v \times b \times \gamma \times m$$

$$15.312 = 55 \times 0.8068 \times b \times \pi \times 0.125 \times 8$$

$$\boxed{b = 110.75 \text{ mm}}$$

5. A 2 teeth right hand worm transmit 2 kW @ 1500 rpm to a 36 teeth wheel. The module of the wheel is 5 mm - pitch diameter of the worm is 60 mm. The normal pressure angle is 14.5° . The Co-eff of friction is found to be 0.06. Determine the center distance. The lead angle and lead. Determine the forces, determine the efficiency of the drive.

Given data:

$$Z_1 = 2$$

$$P = 2 \text{ kW}$$

$$n_1 = 1500 \text{ rpm}$$

$$Z_2 = 36$$

$$m = 5 \text{ mm}$$

$$d_1 = 60 \text{ mm}$$

$$\alpha = 14.5^\circ$$

$$\mu = 0.06$$

$$a = ?$$

$$\phi = ?$$

$$\eta = ?$$

$$\text{Lead } (L) = ?$$

$$F_t = ?$$

$$F_n = ?$$

Solution:

$$i = \frac{Z_2}{Z_1} = \frac{36}{2} \Rightarrow i = 18$$

$$i = \frac{n_1}{n_2} \Rightarrow 18 = \frac{1500}{n_2} \Rightarrow n_2 = 83.33 \text{ rpm}$$

$$d_2 = m \times Z_2 \Rightarrow 5 \times 36 \Rightarrow d_2 = 180 \text{ mm}$$

$$a = \frac{d_1 + d_2}{2} = \frac{60 + 180}{2} \Rightarrow a = 120 \text{ mm}$$

$$\tan \phi = \sqrt[3]{\frac{n_2}{n_1}} \Rightarrow \phi = \tan^{-1} \sqrt[3]{\frac{83.33}{1500}} \Rightarrow \phi = 20.88^\circ$$

$$L = 2\pi m_2 \rightarrow (\text{for double thread worm})$$

(from eqⁿ 12.46 (b) pg 220)

$$L = 2 \times \pi \times 5 \Rightarrow L = 31.45 \text{ mm}$$

$$M_{t2} = \frac{9.55 \times 10^6 \times P \times C_s}{n_2} = \frac{9.55 \times 10^6 \times 2 \times 1.5}{83.33} \Rightarrow M_{t2} = 343.81 \times 10^3 \text{ N}\cdot\text{m}$$

$$F_t = \frac{2M_{t2}}{d_2} = \frac{2 \times 343.81 \times 10^3}{180} \Rightarrow F_t = 3820.11 \text{ N}$$

$$F_n = \frac{F_t}{\cos \alpha \times \cos \gamma} = \frac{3820.11}{\cos(45) \times \cos(20.88)}$$

$$F_t = 4.22 \times 10^3 \text{ N}$$

$$\underline{\underline{\eta = ?}}$$

$$\eta = \frac{\tan \gamma (1 - \mu \tan \gamma)}{\mu + \tan \gamma}$$

$$\frac{\tan(20.88) (1 - 0.06 \times \tan(20.88))}{0.06 + \tan(20.88)}$$

$$\boxed{\eta = 0.84 \%}$$

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